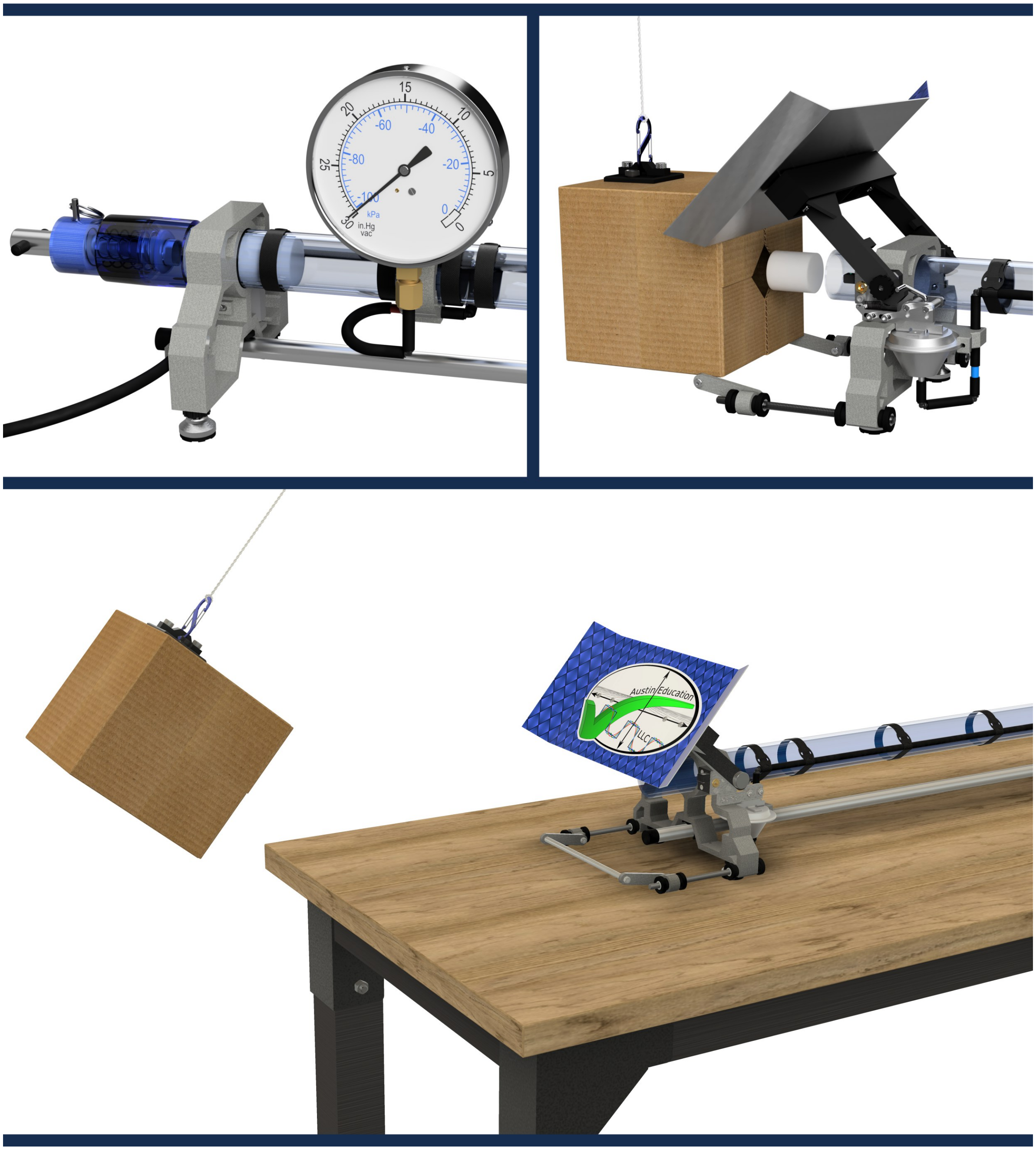
Austin Education LLC

Vacuum Cannon Teacher's Guide



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Contents

1 Introduction

1.1 Purpose of the Teacher's Guide

Welcome to the Teacher's Guide from **Austin Education LLC**. This document provides educators with a wealth of practice problems complete with worked out solutions for the Vacuum Cannon and related STEM topics. These problems can save educators much time in producing problems for homework, in class activities, examples, quizzes, exams or extra credit assignments. The material here is applicable from middle school to advanced introductory undergraduate courses. Recommendations for in class activities and lessons are provided and can be easily adapted for the environment. The User Manual must be read in its entirety before starting any lessons recommended in this guide. The Vacuum Cannon helps engage the students with an exciting display of atmospheric pressure and provides a wealth of physical insight and advancement of students' technical, scientific, and problem solving skills.

1.2 Recommendations, Corrections, and Comments

The Teacher's Guide is a living document and is updated as needed and the latest revision may be downloaded at no additional cost from the website or provided via email.

Please send an email to support@austineducationllc.com if:

1. any grammatical errors or typos are found.
2. any internal or external links are not working.
3. a problem is unclear, ambiguous, or exceeds the difficulty expected for the category.
4. any computational or technical errors are discovered.
5. any solution does not adequately elaborate or show how the problem was solved for the category.

New problem recommendations are welcomed! If you've created a problem for any category that you think would be a good addition to the Teacher's Guide, feel free to send it in by email. An entry in the acknowledgments section will be added (or not if desired). Feel free to send a recommendation for the type of problem you would like to see created including any relevant details. Please place "**New Problem Recommendation**" or similar in the subject line. Comments are welcomed to help improve the completeness and usefulness of this document for the Vacuum Cannon kit.

1.3 How to Use this Document

There is enough material here to be used for lecture based questions or examples, in class problems, activities, homework, laboratory assignments, quizzes, exams, and extra credit. As a rough guide, the complexity of problems are partitioned as follows:

- Introductory: This section covers basic conceptual questions where most of the direction is covered in the question. Basic calculations are emphasized but are largely kept to plug and chug style questions, or when possible, simple derivations are walked through in the problem. Computer based assignments emphasize simple tasks in Excel like taking the average or producing basic plots / charts.

- Intermediate: This section involves more technical conceptual questions resembling those of the introductory section. In general, the mechanics and physics of the Vacuum Cannon are developed more thoroughly. Regular algebraic manipulations are required and steps are provided for some parts to guide students in the right direction. Emphasis on solving equations symbolically and assessing the answer is increased. Basic concepts of calculus are presented but no actual calculus is performed. Students are encouraged to make plots using Excel, Python or any other computer plotting software. Simple online plotters are also recommended such as Desmos.

- Advanced: This section involves a wide range of problems requiring more technically challenging derivations in various disciplines of physics and engineering. Calculus is required regularly with optimization, plotting, and interpreting data with all work solved symbolically. Numerical solution techniques using Python are also introduced. The guidance given varies for derivations. Some of these questions may be usable in the Intermediate section depending on what material the students have been exposed to. Problems requiring the use of Python are left for the students to use the internet as a resource to learn from an abundance of worked examples. Refer to section 5.4 for more information and the "READ ME" document for some additional Python resources.

Fully worked out solutions appear in the blue highlighted text boxes directly following the questions. These are for educators only and not for distribution to students. The solutions here are meant to be sufficiently detailed that the educator can see exactly how the problem was solved and similar demonstrations could be shown to students. All significant stages of the solution (with brief descriptions) are shown for calculations though individual steps for algebraic manipulations and similar operations are omitted for routine work in more advanced problems. Problems where the same process is repeated for multiple input values are also not repeated for brevity. More involved manipulations are demonstrated throughout where required.

The green boxes following the solution provides recommendations, discussions, external resources, historical perspectives, or descriptions of STEM concepts.

1.4 Use Policy of the Teacher's Guide

The intent of this guide is to select questions as fit for your students and tailor them as needed for the curriculum. The following lists what may and may not be done with the Teacher's Guide from Austin Education LLC.

What **MAY** be done with this Teacher's Guide

- Copy, paste, or extract these problems (**but not solutions**) and images in part or in whole and use them in your lesson plans, homework, exams, activities etc. in print or digital form.
- Modify the text, images, equations, language etc. of the problems in any way desired.
- Provide this PDF to other faculty directly in your school.
- Print this document for convenience, provided it is kept in a secure location **not** accessible by students.

What **MAY NOT** be done with this Teacher's Guide

- Provide this Teacher's Guide in part or in whole to the students in digital or print form.
- Upload this document in part or in whole to the internet or media sharing sites in any way.
- Send this Teacher's Guide (digital or print) in part or in whole to anyone outside your school or household. If the Vacuum Cannon is collectively owned and shared between multiple schools or households, the Teacher's Guide may be sent to the secondary school or household.
- Rebrand or resell this guide in part or in whole as a product not associated with Austin Education LLC. Note that the Teacher's Guide from Austin Education LLC is copyright protected.

For almost two decades students have been able to copy and paste homework questions to the internet and find solutions from a variety of sources. While there is no new science unique to this Vacuum Cannon kit, every attempt has been made to produce original problems from scratch covering a spectrum of fields, techniques, and approaches. Most of these problems cannot be directly found online by students which improves the usefulness of the kit as students will need to seek their peers and teachers for help. It requires the collective efforts of the educators who have acquired this Teacher's Guide to not allow this document to be uploaded online. The moment students can access the questions/solutions the effectiveness of the Vacuum Cannon kit is diminished. Please do not remove the password to this PDF. Thanks for your cooperation in keeping the educational content secure.

1.5 Start Here if You're Short on Time

- The User Manual is required for safe operation of the Vacuum Cannon. There is a quick setup video on the **Resources** page of the website to help show the process.
- The Algebraic Worksheet is highly recommended as a first week homework assignment for freshman STEM majors as poor mathematical skills can hinder a physics class. This document is part of the downloadable materials from the website with your access code.
- Section 2 provides the projectile velocity equation.
- The list below gives a few starting recommendations if you need to quickly prepare some questions for an assignment. The list below does not imply these are the best questions but can be a time saver from scanning the entire document.

Conceptual

Introductory: 6.2 - 6.3 - 6.5 - 6.22
Intermediate: 7.2
Advanced: 8.1 - 8.2

Fundamental

Introductory: 6.7 - 6.15 - 6.24 - 6.26 - 6.27
Intermediate: 7.1 - 7.4 - 7.14 - 7.25 - 7.37
Advanced: 8.11 - 8.5

Computational/Excel/Python

Introductory: 6.43 - 6.47
Intermediate: 7.13 - 7.21 - 7.39
Advanced: 8.7 - 8.15

Engineering/Analysis/Optimization

Introductory: 6.32 - 6.50
Intermediate: 7.19 - 7.38 - 7.41
Advanced: 8.4 - 8.6 - 8.10 - 8.12 - 8.13 - 8.17

2 Derivation of the Projectile Velocity Equation

The derivation of the ballistic pendulum velocity equation is a classic example utilizing conservation of energy and momentum. The method shown here is standard but different from historical approaches (and simpler) since we are not measuring the period of an oscillating physical pendulum. The following sections provide the equation in three different ways: directly, briefly derived, and derived in detail with comments. This equation will arise in various forms in the questions that follow. The derivation may be more complex than the grade level taught as it combines conservation of energy, momentum, exponents, trigonometry and of course algebra. The result can be expressed without trigonometric functions if desired. The derivation of the velocity equation is not necessary to proceed using the following questions in the remainder of the Teacher's Guide. Using the final result and having the measured mass of the projectile, box, and swing angle (or height) are all that's needed to determine the velocity of the projectile. The following are developed from Figure 1.

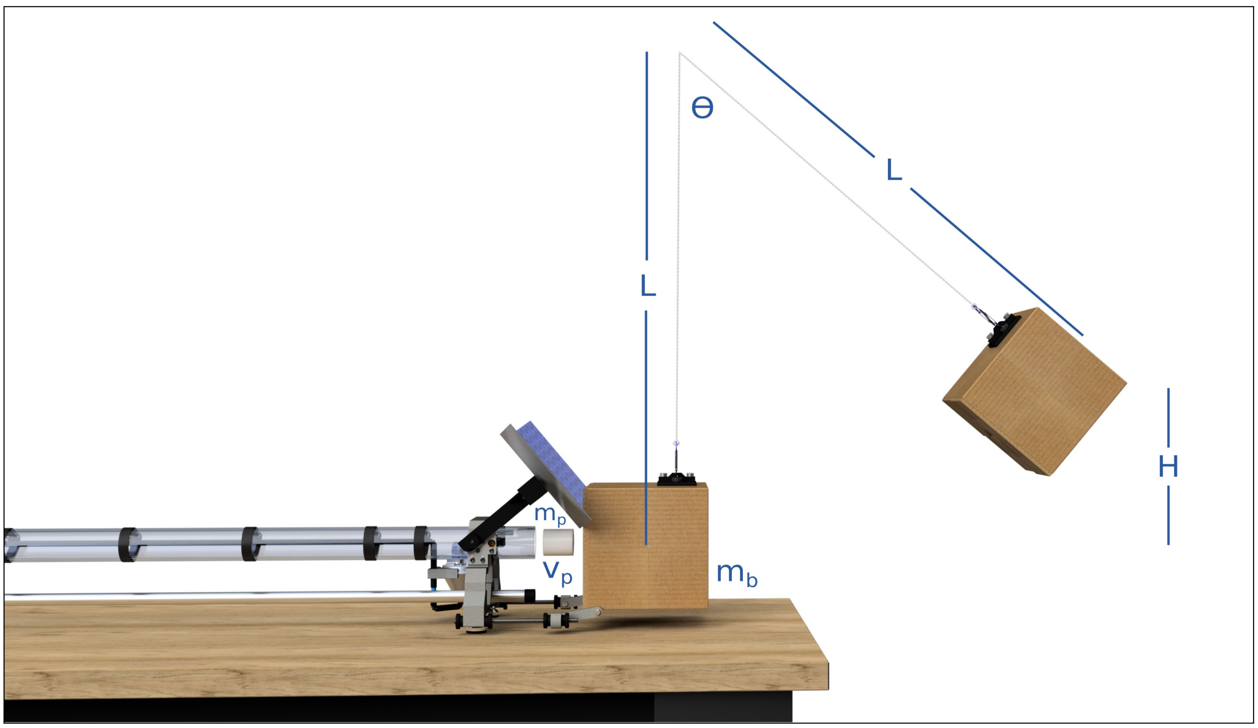


Figure 1: The ballistic pendulum in its standard configuration. The total length of the string and the height H are referenced from the center of the box.

2.1 Projectile Velocity

The length of the string, mass of the projectile, mass of the box, and swing angle must be known to determine the final velocity. The following variables are found in Eq. 1 and 89.

- v_p - Velocity of the projectile in m/s
- m_p - Mass of the projectile in kg
- m_b - Mass of the box in kg
- L - Length of the string in m
- θ - Swing angle of the box in degrees or radians
- H - Height of the box above its starting point (max swing angle)

$$v_p = \left(1 + \frac{m_b}{m_p}\right)\sqrt{2gH} \tag{1}$$

Note that $g \approx 9.8\ m/s^2$ is the acceleration of gravity near the surface of the earth. Since the swing height H can be written in terms of the length of the string and swing angle, Eq. 1 can be written as

$$v_p = \left(1 + \frac{m_b}{m_p}\right)\sqrt{2gL(1 - cos(\theta))} \tag{2}$$

For the best accuracy using the Vacuum Cannon setup, the length L of the string should be measured to the center of the box (See Section 2.4).

The swing angle θ may be used in degrees or radians but students must be shown how to verify which mode their calculator is in, else results will be incorrect. Note that Excel and Python default to radians. All scientific calculators can be swapped to the proper units though the current active unit may not be displayed on the main screen. If the calculator doesn't show, the following is a simple test. If the calculator is in degree mode, then using 60° will give

$$cos(60) = .5 \tag{3}$$

If the calculator is in radians mode, then using the intended 60° will produce

$$cos(60) = -.95241... \tag{4}$$

which indicates the input was 60 rad and not 60° where 1 $rad \approx 57.3°$.

Introductory Section

6.5 Conceptual Question: Pressure 5

Force is related to pressure by the area that the pressure acts on.

$$F = PA \qquad P = \frac{F}{A} \tag{30}$$

The figure below shows two cylinders with different diameters but each has the same mass. Which one results in a larger pressure on the table and why?

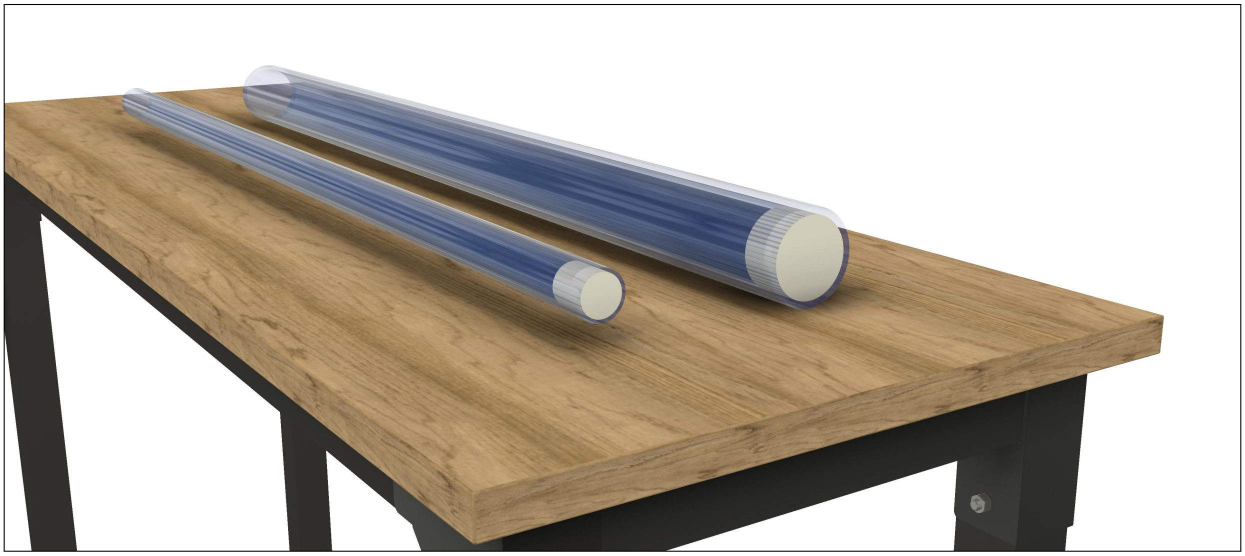
Figure 23: Two cylinders with the same mass (or weight) rest on the table but one has a smaller diameter.

Since both cylinders have the same mass, they both produce the same force (F) on the table. The cylinder on the right has a smaller area (A) contacting the table so the smaller cylinder results in a larger pressure on the table.

This would be a good time to show students a video online of a teacher demonstrating pressure by the classic demonstration of laying on a bed of nails. The force spread out over a larger area reduces the pressure of each nail. This problem might be tailored to include a mass or weight of each cylinder along with a known cross sectional area if it helps students see the result in the numbers first.

6.22 Final Velocity and Projectile Diameter

Imagine you and a friend have lined two vacuum tubes next to each other. They are the same length but your friend built their cannon twice the diameter. When both are launched with the same length projectile, how do you think the final speeds of the projectiles compare? Does your friend's projectile (that's twice the diameter) have the same final speed, faster, or slower? Explain your thinking.



Possible student response:

I think my friend's vacuum cannon with the larger diameter projectile will go faster. They both have the same pressure acting on them but the larger projectile will have more force pressing on it so it will go faster.

These questions seem simple but can be deceptively tricky! The derivation is covered in the Intermediate section. While the projectile that's twice the diameter will indeed experience 4 times the force (4 times the cross sectional area) it will also have 4 times the mass so they end up having the same acceleration, thus the same final speed. This problem helps exercise the student's physical reasoning and at this level should be graded on the effort in explaining their physical thinking. This question is repeated by calculation in the Intermediate section which can be graded on accuracy. While there is a correct answer, "I don't know" is a perfectly acceptable final conclusion. Students should be encouraged to think of how they would make a physical measurement and what factors might affect the results. We make measurements and perform experiments because we don't know what will happen!

6.46 Excel Exercise: Kinematics and Mass Relationship

The following graph shows the position vs. time data for five different mass projectiles obtained via a high speed camera. The projectile masses used were approximately (in grams): 27, 58, 89, 120, 146. Label the plots with the mass projectile you believe it belongs to. Explain your thinking for matching the masses with the plots. What do you think the slope (steepness) of the plots tells you about the projectile?

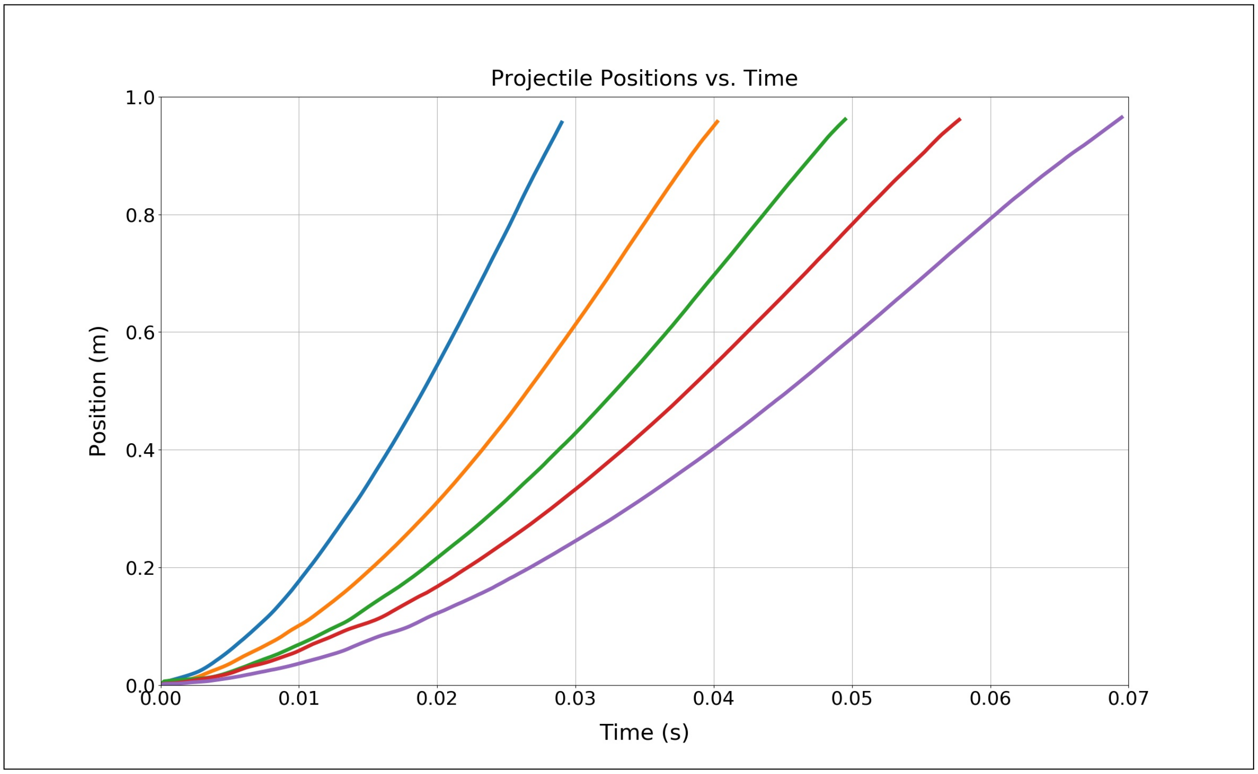


Figure 48: Five different mass projectiles' position data obtained from a high speed camera.

There are a few ways to interpret the graph. Assuming each experienced the same net force, Newton's second law reveals the lightest projectile should experience the greatest acceleration and the heaviest projectile should experience the lowest acceleration. We see the blue line reaches the end point (around .97 m) in a little under .03 seconds. The purple graph reaches the end point in just under .07 seconds, over twice the time. Viewing the 5 plots as such leads us to conclude that the lightest projectile should reach the end in the shortest amount of time. The masses of the projectiles are shown below.

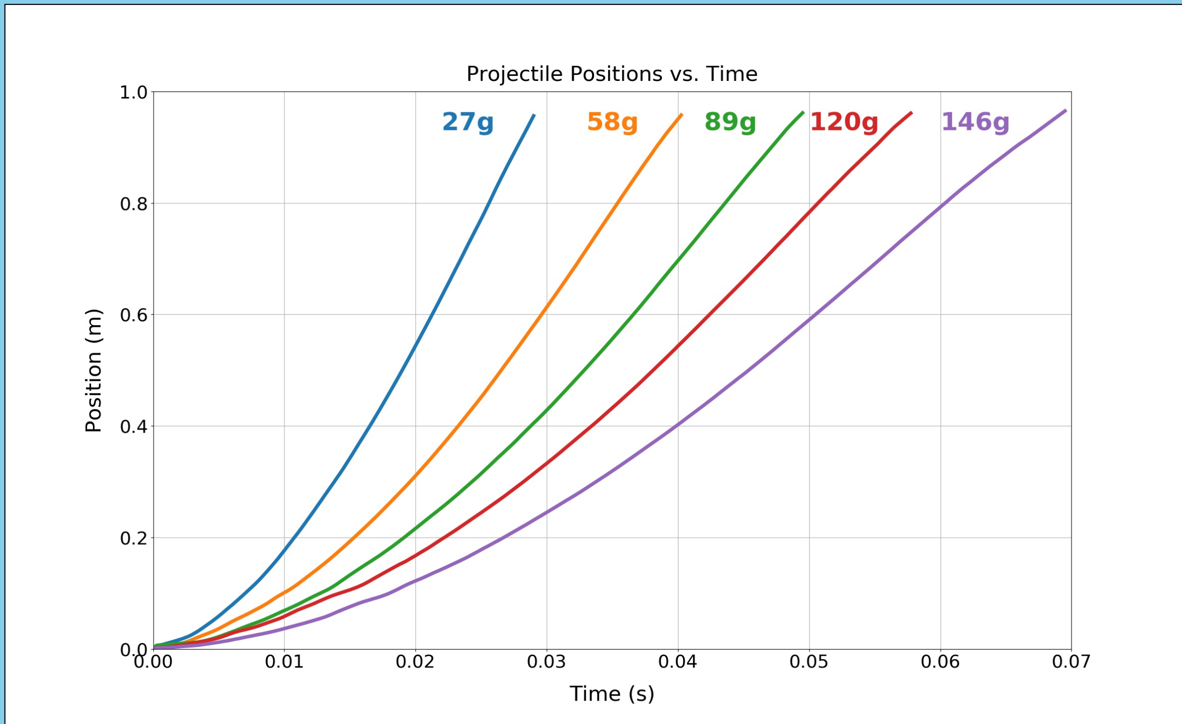


Figure 49: Position vs. time data for five different mass projectiles.

If students observe the shape of the curves carefully (between .6 and .8 m), they will see the slope of the blue curve is steeper than the purple curve. The slope of the curve tells us how fast the projectiles are moving and appears greatest for the blue 27 g curve and lowest for the purple 146 gram curve. The slope of a position vs. time curve gives the velocity. Students will qualitatively and quantitatively learn this in introductory physics courses. When the students take calculus, they will learn that the derivative of position with respect to time produces velocity. **Note: If you want students to plot these for themselves, the Excel file has the projectile columns placed in a random order so it isn't as obvious just looking at the raw data.**

Section 6.47 allows students to plot the data for themselves in Excel and expands upon the "wobbles" shown in the three heavier projectiles. Note that in this problem, the position data has been trimmed to draw more attention to the shapes of the curves during acceleration. The compression of air in front of the projectiles is more noticeable in the data sets used in later problems. The Advanced section provides exercises where students explore higher order polynomial curve fitting to better see how the position changes with time. In introductory physics courses, particularly with kinematics, acceleration is taken as constant to simplify the work which otherwise requires calculus. Many real systems (within limits) may have approximately constant acceleration which simplifies the analysis.

Intermediate Section

7.35 Tension in the String: Static Case

Figure 86 shows the box at its max swing angle. Draw a free body diagram (with coordinate system) and solve for the tension in the string as a function of the variables and constants below. Solve the resulting equation symbolically first and apply some limiting cases to assess if the answer is reasonable. The total length of the string may be considered to the center of the box. Hint: your final result may not have all of the variables below.

- L: Length of the string is 2.5 m
- m_b: Mass of the box is 210 g
- m_p: Mass of the projectile is 70 g
- θ: Swing angle is 46°
- g: Acceleration due to gravity equal to 9.8 m/s^2

Do you think the tension in the string is greater when swinging or at its turning point at the max swing angle? Explain your thinking.

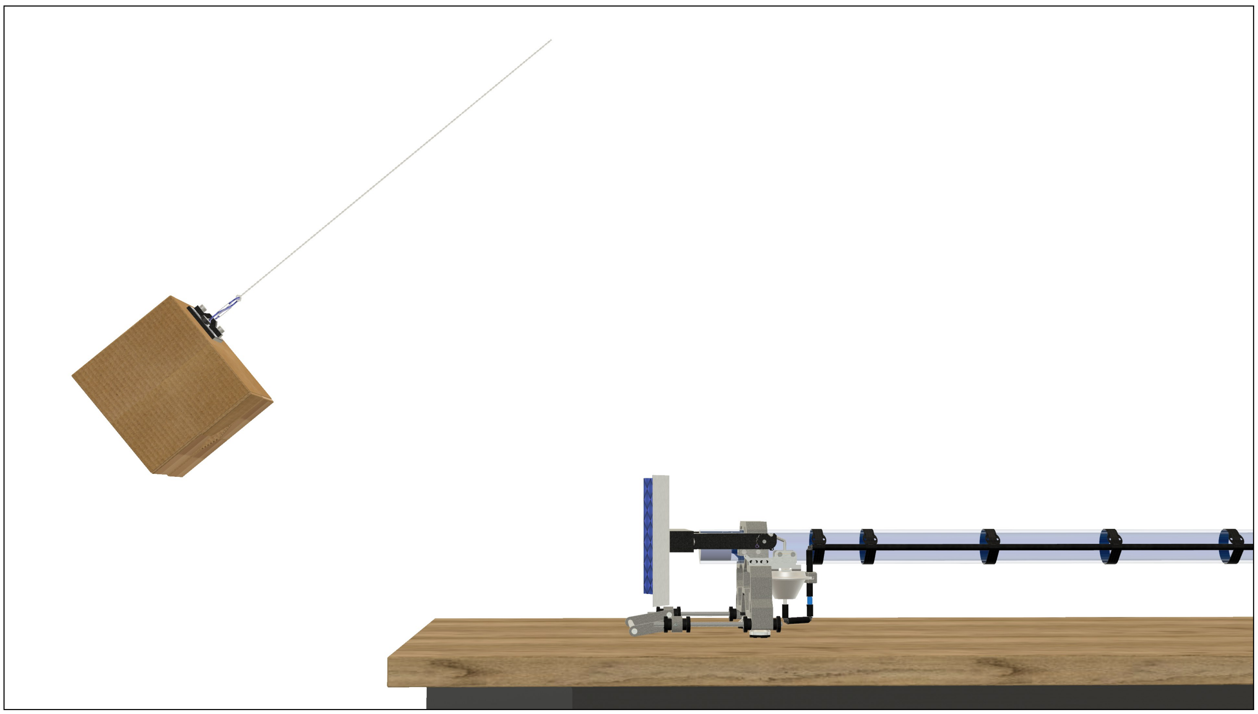


Figure 83: The box is shown at its full swing angle with the projectile inside

The free body diagram is shown below with the box replaced by a dot for simplicity. There are three forces acting on the dot: The force of gravity on the box, the force of gravity on the projectile (which contributes to the mass of the box), and the tension in the string.

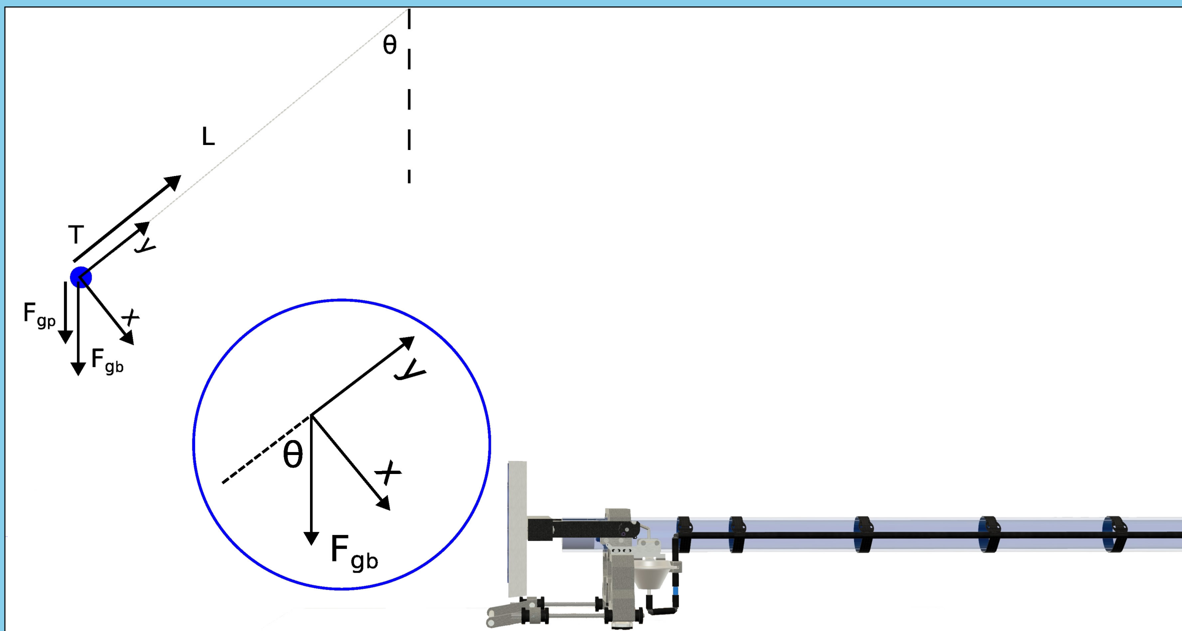


Figure 84: The coordinate system is oriented in the direction of positive centripetal acceleration. A common mistake for students is using the wrong trig function in relation to the angle θ when summing forces.

The $\hat{y}$ direction is toward the center of the swing circle. Since the motion is circular it's best to use centripetal acceleration for the system. Note the exploded view to see the correct placement of the swing angle θ on the coordinate system. Let's define $m = m_b + m_p$ to be the total mass of the projectile and box. Summing the forces in the $\hat{y}$ direction gives

$$F_{nety} = m\left(\frac{v^2}{r}\right) = T - F_{gb}cos(\theta) - F_{gp}cos(\theta) \tag{311}$$

When the box reaches the end of its swing at the largest angle θ, its turning point has been reached and the instantaneous velocity is 0 m/s. Knowing that the force of gravity equals mass times the acceleration of gravity, our equation becomes

$$0 = T - m_b g cos(\theta) - m_p g cos(\theta) \tag{312}$$

Solving for T and factoring the common term of $cos(\theta)$ gives the final tension in the string as

$$T = (m_b + m_p)gcos(\theta) \tag{313}$$

or using our combined mass

$$T = mgcos(\theta) \tag{314}$$

Before inserting numbers, we can try a few limiting cases to assess the answer. The easiest is to consider if the mass of the box and projectile were 0 g. This results in Eq. 314 giving a final tension of $T = 0\ N$, which makes sense as there is no box to support and we neglect the mass of the string. If the instantaneous angle was $\theta = 0°$, Eq. 318 reduces to $T = mgcos(0) = mg$ which is just the weight of the box and projectile (though only if the box is static, which was our condition earlier). Inserting the numbers gives a final tension of $T \approx 1.9\ N$ which is a bit less than half a pound so the answer is reasonable for a cardboard box and plastic projectile.

A common mistake students run into is attempting to solve a problem without drawing out the picture or the coordinate system. Coordinate systems should be required for credit on the problems to discourage students from taking short cuts which often become long cuts when errors arise. If numbers are inserted into the problem before the algebraic solution is complete, students will miss the opportunity to assess if the answer is correct using limiting cases. For example, if the angle θ is placed in the wrong location on their coordinate system, the final result becomes

$$T = (m_b + m_p)gsin(\theta) \tag{315}$$

If the angle in use is 45°, then $sin(\theta)$ and $cos(\theta)$ give the same result. This would produce the correct numerical answer in this scenario, but would produce the incorrect answer in most other systems / setups. Even if the angle is close to 45° the final results will be close. Here, a limiting case can provide a large clue that something is wrong. Because we have stated this system is static (not swinging) we can apply a limiting case for the tension if the box was at the bottom where $\theta = 0°$. The tension should be equal to the weight of the box and projectile but instead gives

$$T = (m_b + m_p)gsin(0) = 0 \tag{316}$$

Since there must be some force of tension with the box at the lowest point, we can assess something is not correct, and eventually work back to the issue of trigonometry. Simply inserting numbers from the start removes this effective problem spotting technique referred to as limiting cases.

Advanced Section

8.5 Projectile Air Drag

The velocity equation gives the velocity of the projectile when it hits the back of the box but not the velocity it leaves the tube at. We can calculate the velocity and position change as a function of time due to air drag.

Figure 100: The projectile is shown just as it exits the tube.

1. Let $x = 0\ m$ be the origin at the end of the tube and air drag is the only force resisting the motion equal to

$$F_d = \frac{1}{2}\rho C_d A v^2 \tag{390}$$

   where $\rho = 1.2\ kg/m^3$ is the air density at 1 atm, $C_d = 1.1$ is the drag coefficient for a horizontal cylinder with a Reynolds number $Re > 10^4$, $A = 9.5 \cdot 10^{-4}\ m^2$ is the cross sectional area of the projectile and v is the velocity. Write out Newton's second law for the projectile (considering horizontal motion and forces only) and arrive at a separable differential equation for $v = v(t)$.

2. Solve the differential equation for $v(t)$ assuming $v(0) = v_0$. Use limiting cases to make sure your result shows that $v \to 0$ as $t \to \infty$ and that $v(0) = v_0$.

3. Integrate your result for $v(t)$ to arrive at $x(t)$, position as a function of time. Use a limiting case to ensure your equation shows that the projectile has traveled 0 m at $t = 0$ seconds.

4. Assuming the projectile has an initial velocity of $v_0 = 33\ m/s$ and hits the back of the box at $x_f = 19\ cm$ away, make plots of $v(t)$ and $x(t)$ using a time range from $t = 0\ s$ to $t = 30\ s$ to show the functions' long term behavior (as if the box were absent). Assume the mass is $m = 28\ g$ and the projectile's diameter is $D = 1.365\ in$.

5. What is the final velocity when the projectile strikes the box and how long does it take to get there? What percentage of the initial velocity is lost by the time it strikes the back of the box? What percentage of the initial kinetic energy is lost? Make plots of $v(t)$ and $x(t)$ over the time range when the projectile strikes the box. Feel free to use Excel, Python, or any available plotter (sketching by hand is acceptable if a straightedge is used for drawing lines).

6. For the purposes of the Vacuum Cannon, is the loss of velocity something students should be concerned about for final velocity results? Explain.

Part 1

The free body diagram only contains the drag force. The force of gravity is neglected as we're only concerned with the horizontal motion and the time of flight is very short.

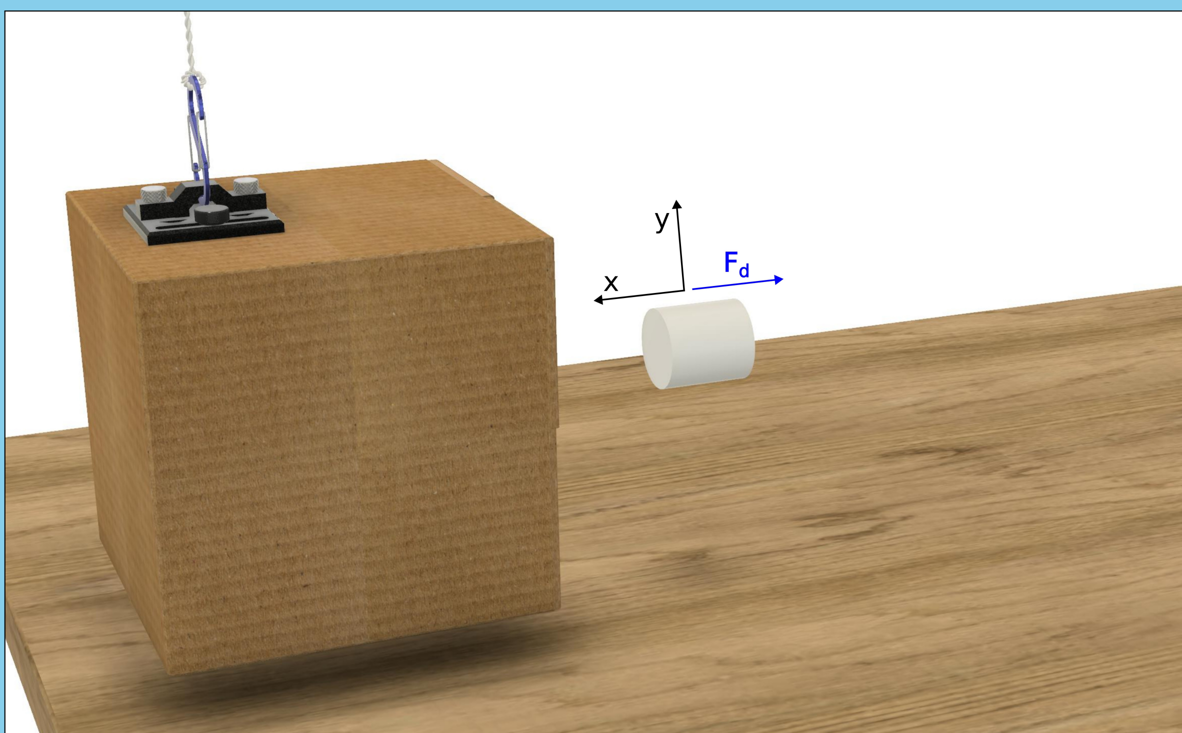


Figure 101: The drag force is the only force with components in the $\hat{x}$.

The only force causing the acceleration in the $\hat{x}$ direction is the drag force so Newton's second law gives

$$m\frac{dv}{dt} = -\frac{1}{2}\rho C_d A v^2 \tag{391}$$

Moving v^2 to the left hand side and moving m to the right hand side, then setting up the integral with respect to t gives

$$\int_{v_0}^{v(t)} \frac{1}{v^2}\frac{dv}{dt}dt = -\int_0^t \frac{\rho C_d A}{2m}dt \tag{392}$$

There are two things we'll do for the next step. First let's define all the constants on the right hand side of Eq. 392 as α to clean up the following work and reduce the amount of writing. Second, we cannot directly integrate the left hand side as is. Instead of "canceling the dt's" (see the notes that follow), we can rewrite the integrand next to v^{-2} using differentials. In general, the differential of a function $v(t)$ is

$$dv = \left(\frac{dv}{dt}\right)dt \tag{393}$$

Using the previous substitution of a differential into Eq. 392 produces the equation ready to integrate as

$$\int_{v_0}^{v(t)} v^{-2}dv = -\int_0^t \alpha dt \tag{394}$$

Part 2

After performing the anti-derivatives we can evaluate as

$$-\frac{1}{v}\Big|_{v_0}^{v(t)} = -\alpha t\Big|_0^t \tag{395}$$

which gives

$$-\frac{1}{v(t)} + \frac{1}{v_0} = -\alpha t \tag{396}$$

and after some algebra we have

$$v(t) = \frac{v_0}{1 + v_0\alpha t} \tag{397}$$

The limiting cases can be applied here to make sure the equation predicts the motion expected for the setup. Letting $t = 0$ gives $v(t) = v_0$ as it should. Letting $t \to \infty$ shows that $v(t) \to 0$ as expected when air resistance eventually slows the projectile asymptotically to 0 m/s. Our simplifying constant can be evaluated as such, otherwise the full equation from variables in the problem statement is

$$v(t) = \frac{v_0}{1 + v_0 \left(\frac{\rho C_d A}{2m}\right) t} \tag{398}$$

Part 3

We can take our expression for $v(t)$ and write it as the derivative of position with respect to time as

$$\frac{dx}{dt} = \frac{v_0}{1 + v_0 \alpha t} \tag{399}$$

We can setup this integral in the same fashion as the previous one as

$$\int_0^{x(t)} \frac{dx}{dt} dt = \int_0^t \frac{v_0}{1 + v_0 \alpha t} dt \tag{400}$$

Using the same type differential substitution for the left hand side and recognizing the right hand side as a derivative of the natural log gives

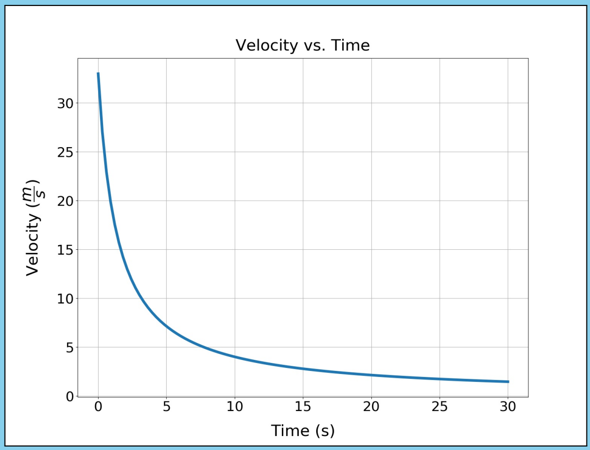
$$x \Big|_0^{x(t)} = v_0 \left(\frac{1}{v_0 \alpha}\right) ln(1 + v_0 \alpha t) \Big|_0^t \tag{401}$$

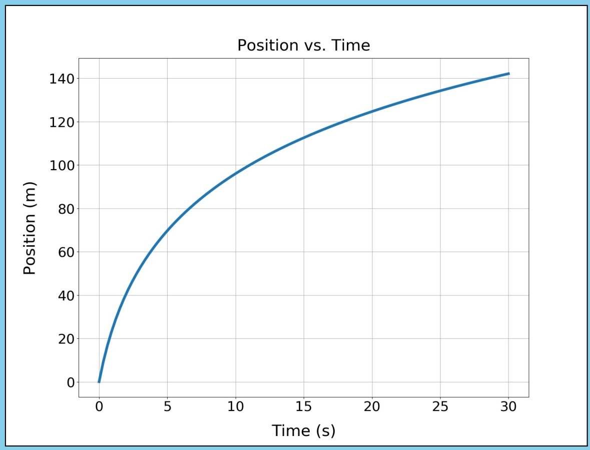
which simplifies to

$$x(t) = \frac{1}{\alpha} ln(1 + v_0 \alpha t) \tag{402}$$

Part 4

To make sure this equation behaves as expected, letting $t = 0$ gives the natural log of 1 which equals zero, thus at $t = 0$ the position of the projectile is $x = 0$ m as expected based on our coordinate system. The plots below show the velocity and position of the projectile as functions of time long after the projectile would have collided with the box. This shows the long term behavior matches expectations (if there was no box).





Part 5

The time of impact is found by solving Eq. 402 for time ($x(t) \to x_f$). Multiplying both sides by α then exponentiating gives

$$e^{\alpha x(t)} = 1 + v_0 \alpha t \tag{403}$$

and solving for t gives

$$t = \frac{1}{v_0 \alpha} \left(e^{\alpha x_f} - 1 \right) \approx .006\ s \tag{404}$$

Using the final result for velocity and inserting the distance to collision gives a decrease in velocity from 33 m/s to 32.86 m/s. The percent decrease is simply

$$\frac{33.00 - 32.86}{33.00} \cdot 100 \approx .4\,\% \tag{405}$$

which is well under a 1% change. The easiest way to determine the percentage of initial kinetic energy lost is by

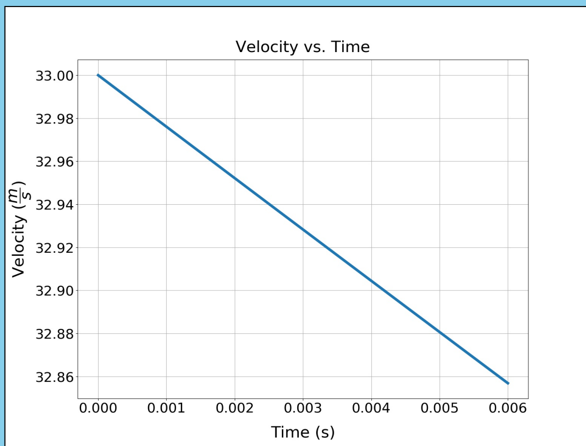
$$\frac{\frac{1}{2} m v_0^2 - \frac{1}{2} m v_f^2}{\frac{1}{2} m v_0^2} \cdot 100 \tag{406}$$

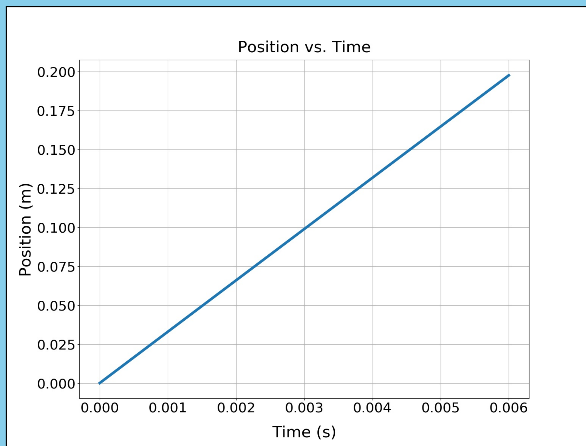
or after simplifying

$$\left[1 - \left(\frac{v_f}{v_0} \right)^2 \right] \cdot 100 \approx .85\% \tag{407}$$

Note carefully the decimal as this is still under 1 %

The graphs below show the results up to the time of collision. Under short time intervals the change of velocity and position are approximately linear which is true for any function examined on a short enough time scale. This is the bases for linearization which approximates a function around a given point.





Part 6

The change in velocity from the time the projectile leaves the tube and collides with the box is negligible for the Vacuum Cannon as a demonstration kit. When using such a setup historically for accurate projectile velocities, one could compensate for all known sources of error (better term, uncertainty) as far as desired. A change of less than 1% makes little difference in the final results, which have multiple sources of error. It would be difficult obtain reliable results with less than 1 % error particularly how the angles are measured for the Vacuum Cannon demonstration. Air drag on the box has a significant influence also (see Section 8.17).

Most real fluid flows quickly require numerical techniques when eliminating simplifying assumptions. A problem of this type can be done analytically to show that there is minimal decrease in velocity by the time the projectile hits the box. Students will learn that careful attention to the coordinate system can greatly reduce the headaches when solving problems. For example, if the coordinate for positive $\hat{x}$ was reversed, the initial condition would be $v(0) = -v_0$.

When working toward setting up a separable differential equation students are often taught (or it is brushed over) that the differential quantities can be treated as fractions. For example, in the following it is often stated that the "$dt's$" cancel.

$$\frac{dy}{dt}dt \rightarrow dy \tag{408}$$

While the definition of a derivative as a limit is developed from a fraction (ratio), the relation

$$\frac{dy}{dx} \tag{409}$$

is **not** in general a quantity "dy" divided by "dx". The notation of

$$\frac{d}{dx} \tag{410}$$

is a differential operator that is waiting to operate on the function $y(x)$, which is often just written as y.

$$\frac{d}{dx}y(x) \rightarrow \frac{d}{dx}y \rightarrow \frac{dy}{dx} \tag{411}$$

Suppose we have a separable differential equation in the form of

$$N(y)\frac{dy}{dx} = M(x) \tag{412}$$

Here, both sides are normally multiplied by dx and the equation is ready to integrate. This is a shortcut on notation that produces the same result as substituting in the proper differential. We can see where this comes from by first considering the standard equation of a line with slope m as

$$y = mx \quad \rightarrow \quad y = \left(\frac{\Delta y}{\Delta x}\right)x \tag{413}$$

We can see the same form for the differential dy as

$$dy = \left(\frac{dy}{dx}\right)dx \tag{414}$$

If we go to integrate both sides of Eq. 412 with respect to x we get

$$\int N(y)\frac{dy}{dx}dx = \int M(x)dx \tag{415}$$

and can see the differential relation for dt from Eq. 414 which then becomes the dy students get by the shortcut method of "cancelling the dx's". Students getting too comfortable with thinking of this as a fraction will eventually run into trouble with multidimensional systems.

Thanks for choosing

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